X_a^0 = mole fraction of reacting gas at $\bar{x} = 0$
$X_a(I+1, J \pm 1) = X_a(\theta + \Delta\theta, \bar{x} \pm \Delta\bar{x})$
θ = dimensionless time, $= tI_0/(h_{av}\, nFC)$

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Pressure Drop and Holdup in Horizontal Slug Flow

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The Dukler-Hubbard slug-flow model is used along with independent correlations for in situ holdup and slug frequency to predict pressure drops for two-phase slug flow. The holdup and frequency correlations are for the most part based on data for air-water flowing in a 1.50-in. pipe. Predictions of pressure drop using this approach are compared with experimental data taken from studies utilizing various systems and pipes from 1.50 to 6.065 in. in diameter. Alternative correlations by Dukler and Hughmark for predicting two-phase pressure drops and holdup, respectively, are included for comparison. In general, the present approach is at least equivalent to the Dukler-Hughmark method, and for values of $\Delta P/L > 0.06$ lb./sq. in./ft. appears to be slightly better.

The simultaneous flow of gas and liquid in pipes is frequently encountered in refineries, chemical plants, and pipelines. For designing these systems, reliable techniques are needed for predicting liquid holdup and pressure drop for gas-liquid flows under various conditions. This paper concerns the prediction of holdup and pressure drop for one of the more common and important situations—two-phase slug flow in horizontal pipes.

Early two-phase flow studies emphasized the development of overall pressure drop and holdup correlations encompassing all types of flow regimes. Furthermore, most of the experimental data were obtained from relatively small and short pipes. Lockhart and Martinelli (*13*) developed one of the first general correlations. Although various other general correlations have since been proposed, the original Lockhart-Martinelli approach is still in many respects the best, as discussed by Dukler et al. (*4*); however, expected errors in predicted pressure drops are on the order of ±30% or more. The general correlation procedures yield fair predictions of pressure drop for all flow regimes because they are based on a large amount of correlatable data. However, when these correlations are applied to systems other than those used in their development, or to flow over extended distances (definitely established flow), predicted pressure drops can be in error by as much as a factor of 2. For more reliable predictions of pressure drop, correlations based on specific models for individual flow regimes are preferable.

More recently, a number of frictional pressure drop correlations have been developed through similarity analysis by Dukler et al. (*4, 5*). A comparison of pressure-drop predictions with a large number of data points showed that case II, based on a model that assumed homogenous flow with slip between the phases, was superior to all other cases as well as to other pressure-drop correlations tested. This correlation is the most reliable published general correlation for predicting frictional pressure drop (*1*). In order to use the Dukler et al. correlation, an accurate estimation of in situ holdup is necessary. Dukler et al. (*4*) tested several available holdup correlations with a substantial number of data points and concluded that the Hughmark correlation (*11*) was clearly the best. Therefore the Hughmark holdup correlation combined with the Dukler et al. equation (case II) represents the best general pressure-drop prediction method available [see sample calculation by Anderson and Russell (*1*)]. This combination will be referred to hereafter as the Dukler-Hughmark method.

This paper describes an alternative to the Dukler-Hughmark method for the particular case of horizontal slug flow. The present approach utilizes a slug-flow model developed by Hubbard and Dukler (*10*), a holdup correlation

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discussed recently by Greskovich et al. (*6*), and a new slug frequency correlation. The two methods are compared using experimental slug-flow data taken from the literature.

HOLDUP PREDICTION

The Hughmark holdup correlation can yield uncertain holdup values for intermittent flows, especially near the stratified flow-intermittent flow transitions. The deviations from experimental data are largest at relatively low flow rates and at low liquid qualities. In a previous paper, Greskovich et al. (*6*) presented a graphical correlation of in situ holdup involving only the mixture Froude number and input liquid quality. This approach, originally proposed by Mamayev (*16*) and Guzhov et al. (*7*), was tested by Greskovich et al. with holdup data for air-water flow collected during a study with an 80-ft. long, 1.50-in. I.D. pipe. The curves in Figure 1 represent the experimental data well, notably at the intermittent flow-stratified flow transitions corresponding to the S-shaped regions in the curves (*6*). Figure 2 compares values of in situ holdup from Figure 1 with those predicted by the Hughmark correlation (*11*); the holdups predicted by the latter correlation are as much as 30% below the values from Figure 1. The largest deviations occur at the transition regions for the various mixture Froude numbers.

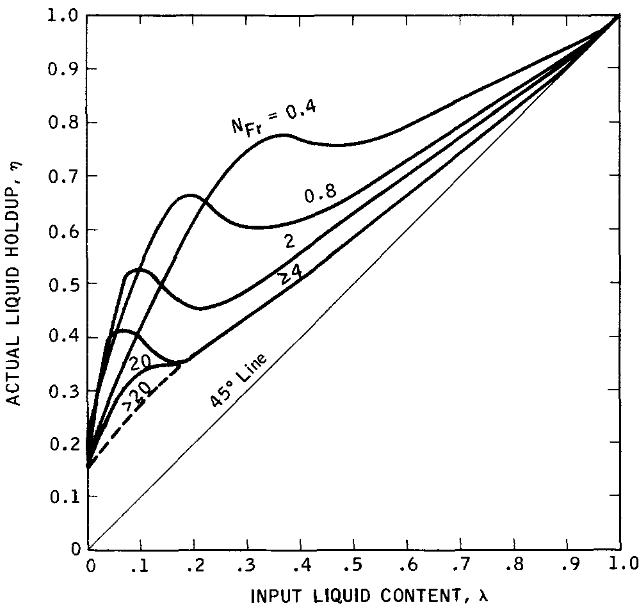

Fig. 1. Dependence of actual liquid holdup on input liquid content and mixture Froude number for horizontal gas-liquid flows.

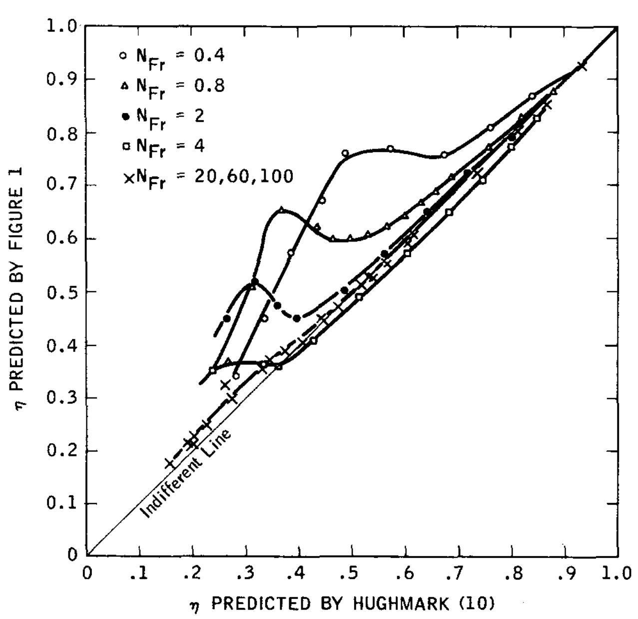

Fig. 2. Comparison of actual liquid holdup prediction techniques for horizontal air-water flow.

The graphical correlation (Figure 1), which is very simple to use, has been incorporated in the present development to estimate in situ holdup for several systems other than air-water (see Table 1). However, the validity of the simple graphical correlation technique for predicting holdup in these systems still needs to be established with experimental holdup data. In practice, the approach proposed below for calculating slug-flow pressure drop would utilize the most reliable holdup correlation available.

TABLE 1. EXPERIMENTAL SLUG-FLOW PRESSURE-DROP DATA

Investigator(s)	System	Pipe diameter, in.	Pipe length, ft.	System pressure, lb./sq. in. abs.
Greskovich & Shrier	Air-water	1.5	80	~14.7
Reid et al. (*16*)	Air-water	4.026	~76	25 to 30
Reid et al. (*16*)	Air-water	6.065	~76	25 to 30
Greskovich & Shrier	Air-water	6.065	100	~14.7
Andrews & Brown (*2*)	Natural gas-water	2.065	1875	~350 to 800
Andrews & Brown (*2*)	Natural gas-distillate	2.065	1875	~350 to 800
Andrews & Brown (*2*)	Natural gas-crude oil	2.065	1875	~350 to 450
Andrews & Brown (*2*)	Natural gas-water	2.065	1875	~150 to 400

SLUG-FLOW MODELS

Two-phase slug flow in horizontal pipes has received attention in the past mainly to provide design engineers with the means to avoid this flow regime. Unlike the case of vertical two-phase flow there is somewhat more flexibility in designing for horizontal pipes. Although the slug-flow regime has been included in experimental studies, specific details of flow phenomena for this regime are scanty. Moreover, much of the slug-flow data reported in the literature are of limited usefulness for developing a slug-flow correlation, primarily due to the short lengths of pipe employed. Since slug flow is generated by instabilities, the various literature data must be carefully evaluated to ensure that they represent established flow conditions. In most of the cases reported, such an assessment is difficult if not impossible.

Published studies covering a detailed analysis of and model development for slug flow include those of Kordyban (*13*), Kordyban and Ranov (*14*), Hubbard (*9*), Hubbard and Dukler (*10*), Dascher (*3*), and Oliver and Wright (*17*). Oliver and Wright were primarily concerned with heat transfer, and slug-flow details are secondary. The original work of Kordyban (*13*) was based on steam-water data collected in 0.315-in. I.D. and 0.420-in. I.D. tubes 6 ft. long. Kordyban's slug-flow model assumes that the liquid slug slips over a slower moving film of liquid at approximately the velocity of the gas. The subsequent paper by Kordyban and Ranov (*14*), based on air-water data collected in a 10 ft. long 1.25-in. I.D. tube, included slug frequency and slug velocity data collected to develop the model further. Kordyban and Ranov point out that the original model of Kordyban does not describe satisfactorily the slug-flow behavior because it was based on insufficient data. Subsequently many independent studies, especially those using dye injection techniques, have shown that the slugs do not move over a liquid film but move through the liquid film.

An extensive program at the University of Houston (*3, 9, 10*) has led to the development of a slug-flow model which assumes that the pressure drop associated with a slug moving through a pipe results from a combination of frictional losses at the wall and acceleration losses due to liquid exchange between the slower moving liquid film and the liquid slug. The model has been tested by Hubbard (*9*) with slug-flow data for air-water in a 1.5-in. I.D., 65-ft. long glass pipe. The remarkably good agreement of experimental pressure-drop data with predictions using the slug-flow model was discussed in a recent presentation (*10*). The major inadequacy in the original development (*9, 10*)—the lack of slug and film holdup data—was partially overcome in the subsequent work of Dascher (*3*). In this study a technique was developed for measuring the holdups. This was accomplished by weighing part of the slug or film by floating part of the pipe and attaching a force transducer to a weighing cell. Dascher concluded that the experimental data collected in his study may be somewhat questionable, since using his results with the Dukler-Hubbard model yields values of frictional and acceleration pressure drops which conflict with those of Hubbard (*9*). However, the total slug pressure drops predicted by the model in both studies agree closely with the experimental values.

DEVELOPMENT OF THE DUKLER-HUBBARD SLUG-FLOW MODEL

The Dukler-Hubbard model permits predicting horizontal slug-flow pressure drops if slug frequency, slug length, and slug and film holdups are known or can be estimated separately. For lack of more reliable film and slug holdup data, Dascher's results are used in the present work; then what remains is to predict slug frequency and slug length and to sum the pressure drops over all the slugs in the given length of pipe. If a reliable estimate of total in situ liquid holdup can be made, then it is sufficient to know either the slug lengths or slug frequency to use the model.

In this paper, the Dukler-Hubbard model is combined with a holdup correlation previously prepared by Greskovich et al. (*6*) and a new slug frequency correlation to yield a useful pressure drop prediction method. A substantial number of data points from various studies are used to test the final pressure-drop predictions. For horizontal slug flow the present method appears to be at least equivalent to or slightly better than the Dukler-Hughmark method.

Hubbard's model (*9*) postulates slug flow to resemble the movement of a gravity wave. In pipe flow, however, the liquid bridges the pipe and the slug behaves similarly to ordinary single-phase turbulent pipe flow. Furthermore, in established flow there is uniform movement into and out of the slug. This is supported by dye injection tests showing that the slug moves through the dye, and by the observation that slug lengths are relatively uniform for a given set of conditions. Due to the movement of liquid into and out of the slug, the actual liquid velocity and apparent translational slug velocity are not equal. The actual rate of advance of the liquid slug is denoted by V_p (indicating the velocity of a fluid particle within the slug), while the translational slug velocity is denoted V_t.

The total pressure drop over a liquid slug moving in a pipe was given by Hubbard (*9*) as follows:

$$\Delta P_s = \Delta P_f + \Delta P_a \tag{1}$$

where

$$\Delta P_f = \frac{2 f_{tp} \rho_L \eta_s (V_{ns})^2 L_s}{144 D g_c} \tag{2}$$

$$f_{tp} = 0.0014 + 0.125/N_{Re}^{0.32} \tag{2a}$$

$$N_{Re} = \frac{D V_{ns} \rho_L}{\mu_L} \tag{2b}$$

and

$$\Delta P_a = \frac{X}{144 g_c A}(V_{ns} - V_f) \tag{3}$$

Through a mass balance, the liquid pickup X is given by

$$X = \rho_s A V_p \left(\frac{V_t}{V_p} - 1\right) \tag{4}$$

It was theoretically shown by Hubbard that the ratio of V_t to V_p for a slug is ~ 1.25. [A similar value can be deduced from the works of Griffith and Wallis (*7*) and Hughmark (*12*).] Experimentally, Hubbard obtained values from ~ 1.25 to 1.28. Furthermore, based on the slug-flow model, the continuity equation leads to the conclusion that $V_p = V_{ns}$. Hence

$$X = \eta_s \rho_L A V_{ns} c \tag{5}$$

where

$$0.25 < c < 0.28$$

It was further shown by Hubbard that the liquid holdup in the film and slug can be related by the following equation:

$$\eta_f = \eta_s \left[\frac{c}{(1 + c) - V_f/V_{ns}}\right] \tag{6}$$

Equations (3), (5), and (6) are combined to give the following equation for the acceleration pressure drop:

$$\Delta P_a = \frac{\eta_s \rho_L c^2 (V_{ns})^2}{144 g_c}\left[\frac{\eta_s}{\eta_f} - 1\right] \tag{7}$$

Let there be N slugs in an extended length of pipe L_p. Then the slug frequency can be expressed as

$$\nu_s = \frac{N}{L_p}(V_t) \tag{8}$$

where

ν_s = slug frequency, sec.$^{-1}$

Recalling that $(V_t/V_{ns}) = (1 + c)$, one can rewrite the expression for slug frequency as

$$\nu_s = \frac{N}{L_p}(1 + c) V_{ns} \tag{9}$$

The pressure drop over the section of pipe L_p is written as

$$\left(\frac{\Delta P_T}{L_p}\right) = \frac{N}{L_p}\Delta P_s \tag{10}$$

Making the assumption of equal slug lengths for a given set of flow rates [this is supported by the work of Kordyban and Ranov (*14*), Hubbard (*9*), and the authors] permits combining Equations (1), (2), (7), (9), and (10) to yield the following form for the total pressure-drop gradient:

$$\left(\frac{\Delta P_T}{L_p}\right) = \frac{2 f_{tp} \rho_L \eta_s (V_{ns})^2 N L_s}{(144) D g_c L_p} + \frac{N \eta_s \rho_L c^2 (V_{ns})^2}{(144) g_c L_p}\left(\frac{\eta_s}{\eta_f} - 1\right) \tag{11}$$

If an accurate value for the total in situ holdup η can be obtained, the product NL_s in the first term of Equation (11) can be calculated by making a liquid balance in any given pipe section:

$$\eta L_p = N \eta_s (L_s) + (L_p - N L_s) \bar{\eta}_f \tag{12}$$

The assumption is now made that η_f is not only the liquid holdup in the film just prior to slug pickup, but also is approximately equal to $\bar{\eta}_f$ (the average holdup in the film between the slugs). High-speed photographs taken by the authors show that the holdup in the liquid film becomes established extremely fast after passage of the slug, as a result of a rapid deceleration of the liquid leaving the slug. Accordingly, Equation (12) can be rewritten as

$$NL_s = L_p \frac{(\eta - \eta_f)}{(\eta_s - \eta_f)} \tag{13}$$

Now, upon substituting Equations (13) and (9) into the first and second terms, respectively, in Equation (11) and simplifying, one can write the pressure-drop gradient for a section of pipe L_p as

$$\left(\frac{\Delta P_T}{L_p}\right) = \frac{\rho_L \eta_s N_{Fr} g}{144 g_c}\left[2 f_{tp}\left(\frac{\eta - \eta_f}{\eta_s - \eta_f}\right) + \frac{\nu_s c^2 D}{(1+c) V_{ns}}\left(\frac{\eta_s}{\eta_f} - 1\right)\right] \tag{14}$$

where

$$N_{Fr} = \text{mixture Froude number, } (V_{ns}^2)/Dg$$

In order to use Equation (14) successfully for a given gas-liquid system in slug flow, the following quantities must be obtained: (1) In situ holdup. This can be estimated from Figure 1. (2) Liquid film and slug holdups. The data presented by Dascher (*3*) are used in the present work. (3) Slug frequency. Several investigators have reported slug frequency data, as discussed below.

Since the mixture Froude number was found to be a satisfactory parameter for correlating in situ holdup, the experimental data of Dascher (*3*) were replotted using mixture Froude number as a parameter (Figure 3). For lack of additional data, it is assumed that Figure 3 can be used for scale-up purposes even though all the data were collected with air-water in a 1.50-in. I.D. pipe.

Slug frequency data that have appeared in the literature, for example, those of Kordyban and Ranov (*14*) and Hubbard (*9*), show a good deal of scatter when plotted against the gas rate with liquid rate as a parameter or vice versa. Furthermore, when plotted in this manner the data from the different sources are not directly comparable, except with regard to the similar trends they exhibit, that is, distinct minima in the curves. However, if slug frequency data are plotted against the mixture Froude number using the flowing liquid fraction as a parameter, the data can be correlated as shown in Figure 4. These data were collected in pipes ranging from 1.25 to 6.065 in. I.D., and represent experiments with air-water and nitrogen-kerosene systems. This method of plotting slug frequency data appears to provide a satisfactory basis for scaleup and is tentatively accepted as valid in the present development. Additional data would be desirable to substantiate this correlation procedure further.

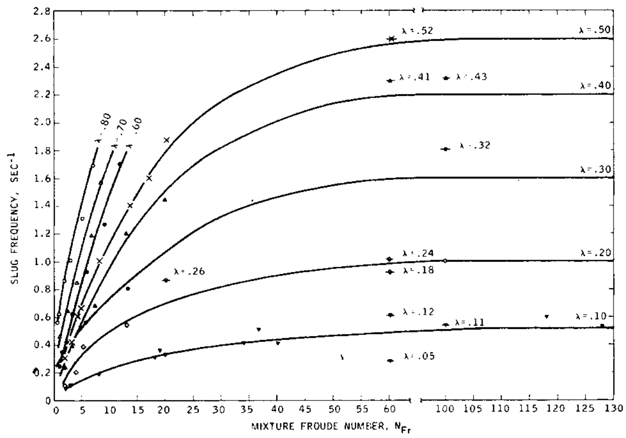

Fig. 4. Slug frequency correlation. Source of data: present work, 1.50-in. pipe, nitrogen-kerosene. Present work, 1.50-in. pipe, air-water. Present work, 6.065-in. pipe, air-water. Hubbard, 1.50-in. pipe, air-water. Kordyban and Ranov, 1.25-in. pipe, air-water.

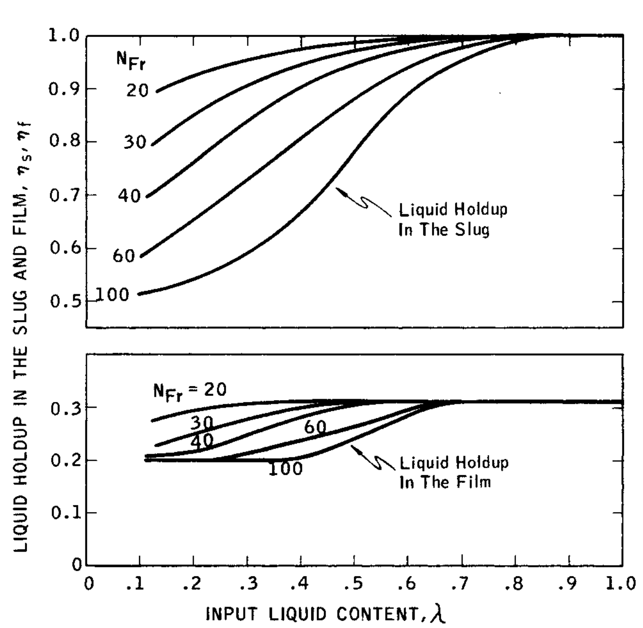

Fig. 3. Liquid holdup in the slug and film for slug flow in a 1 ½-in. pipe.

RESULTS

By combining Figures 1, 3, and 4 and Equation (14), the pressure drop for a given pipe size and gas and liquid flow rates can be calculated. In the present work the theoretical value of 0.25 for c was chosen. Because Hubbard's data (*9*) indicate that this value could be as great as 0.28, the second term in Equation (14) may yield values that are too low by as much as 18%. However, since the values of the acceleration pressure drops were found to be at most equal to the frictional pressure drop in the range of no-slip velocities up to approximately 10 to 15 ft./sec., and less than the frictional pressure drop at velocities greater than these, the overall relative error introduced in the calculated pressure drops would be no greater than approximately 9%, that is, the calculated values could be too low by 9% as a result of using the theoretical value of c.

The two-phase friction factor can be calculated by Equations (2*a*) and (2*b*). Since the liquid holdup within the slug is on the average greater than 0.7, the flow of a slug in a pipe can be approximated by a homogeneous model with the slug experiencing a smooth liquid film at the wall. Therefore the Reynolds number, Equation (2*b*), incorporates the liquid density and viscosity. This point is more completely described by Dukler et al. (*5*).

Pressure drops calculated using Equation (14) and Figures 1, 3, and 4 were compared with experimental data collected by various investigators (Figure 5). The sources of experimental data are given in Table 1. Data collected at the University of Houston were excluded from this comparison since these data were used to develop the model. The experimental air-water data collected by the authors using an 80-ft. long, 1.50-in. I.D. pipe are believed to be reliable and to represent established slug flow. The agreement of the calculated values for pressure drop with the experimental values was relatively good, especially at the larger values of $\Delta P/L$ (>0.06 lb./sq. in./ft.)

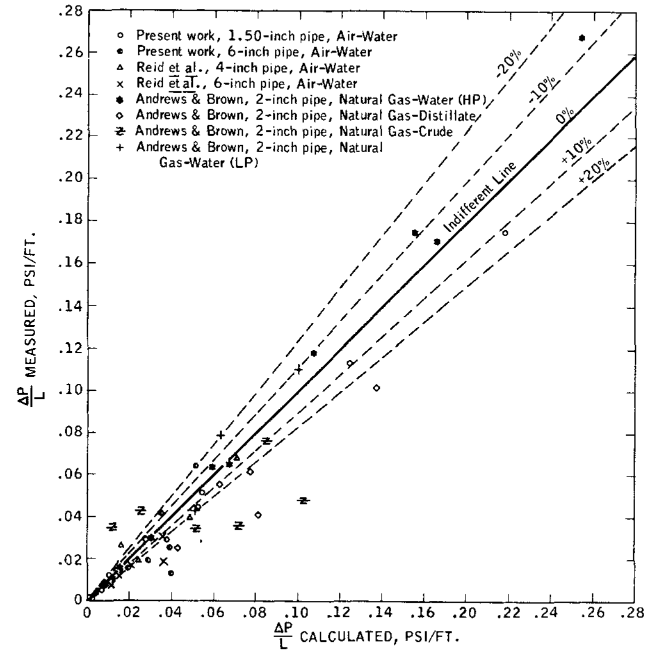

Fig. 5. Calculated versus measured ΔP's using Dukler-Hubbard model and correlations presented.

In order to examine the effect of pipe size on the prediction method, a limited number of experimental data points for the air-water system collected by the authors in a standard 6-in. commercial pipe, as well as other air-water data collected in standard 4- and 6-in. commercial pipes by Reid et al. (*18*), were also checked. In both cases it is believed that established slug flow was not completely attained due to an insufficient entrance length. In the authors' work the pipe length was approximately 100 ft. and in that of Reid et al. the pipe length was approximately 75 ft. It was found that the calculated pressure drops were for the most part within ±25% of the measured values, but the majority of calculated values were greater than the measured values. The Dukler-Hubbard model has the property that if established slug flow is not attained, the acceleration pressure losses are very low due to very little actual liquid exchange. Therefore the model predicts larger values of acceleration pressure drop than actually exist.

The effect of fluid properties on the prediction method was tested using data from the work of Andrews and Brown (*2*). In this study tests were run in a 2-in. steel pipe approximately 1,689 ft. long with water, distillate, crude oil, and natural gas. The data selected for evaluation were taken approximately 550 ft. from the inlet. The pressures at this point were in the neighborhood of 200 to 500 lb./sq. in. gauge. On the whole, the natural gas-water data agreed within ±10% with the predicted values, while the pressure-drop predictions for the natural gas-distillate and the natural gas-crude oil systems showed increasing deviation when compared with measured values.

For comparison, the Dukler-Hughmark correlations (*1*) for predicting ΔP and η, respectively, were also evaluated. For the data from the studies summarized in Table 1, pressure drops were calculated and are compared with the measured values in Figure 6. For the larger values of ΔP, the predicted pressure drops are generally greater than those predicted by the method proposed in the present work. The relative accuracy of the two methods is more readily seen in Figures 7 and 8, which present arithmetic mean and standard deviations between predicted and measured pressure drops for both methods over discrete ranges of $\Delta P/L$ values. The arithmetic mean deviation $\bar{d}$ is calculated by the following equations:

$$d_i = \left| \frac{P_i - M_i}{M_i} \right|$$

$$\bar{d} = \left(\sum_{i=1}^{n} d_i \right) \Big/ n$$

and the standard deviation σ is defined as

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (d_i - \bar{d})^2}{n-1}}$$

where P_i = predicted ΔP; M_i = measured ΔP; n = number of data points.

It was convenient to select intervals of pressure drop to evaluate the deviations for the two methods. As shown in Figure 7 for the lower values of the total pressure drop gradient $\Delta P/L$, the Dukler-Hughmark correlations yield

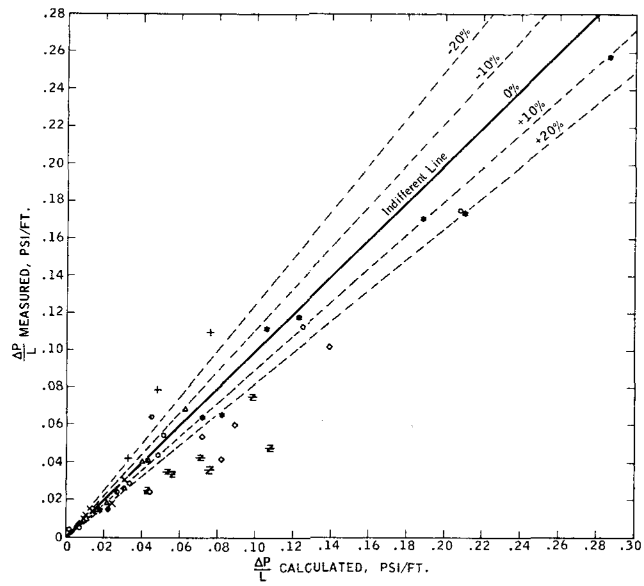

Fig. 6. Calculated versus experimental ΔP's using Dukler-Hughmark pressure-drop and holdup correlations (data designation same as Figure 5).

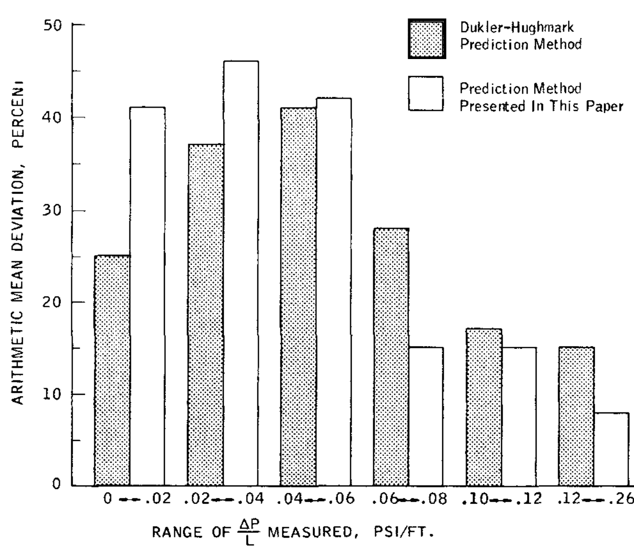

Fig. 7. Arithmetic deviations between predicted and measured pressure drops.

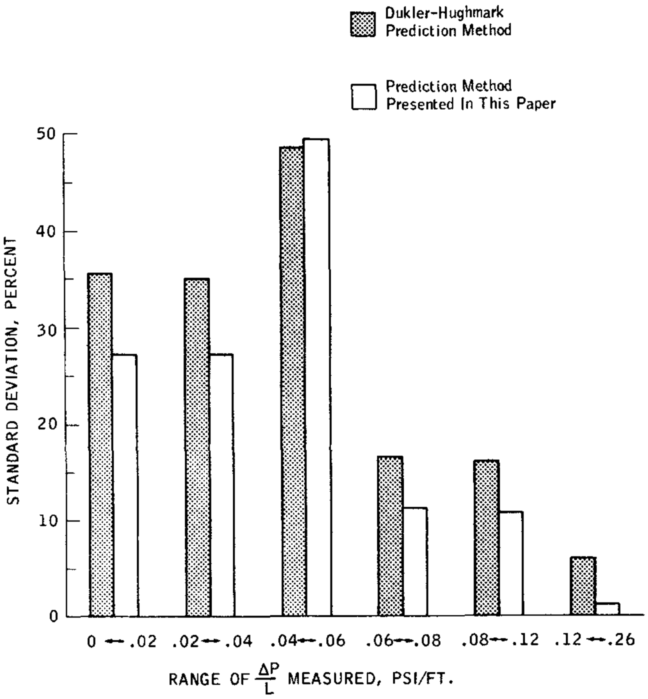

Fig. 8. Standard deviations between predicted and measured pressure drops.

predicted values with relatively lower arithmetic mean deviations than the present method. For values of $\Delta P/L$ of ~ 0.04 to 0.06 lb./sq. in./ft., the two methods are essentially equivalent. At values of $\Delta P/L > 0.06$ lb./sq. in./ft., the present method appears more reliable. This is further illustrated by the comparison of standard deviations in Figure 8 which suggests that the present method is on the whole slightly better than the Dukler-Hughmark method. For the majority of the data considered in Figures 5 and 6, the mixture Froude number was approximately 4 or greater. Thus it can be noted from Figure 2 that for $0.20 < \eta < 0.60$, the Hughmark holdup correlation (*11*) yields holdups for the air-water system within 10 to 15% of those from Figure 1. For slug flow at Froude numbers less than 4, the holdups predicted by the Hughmark correlation could be more than 20% lower than actual values as shown in Figure 2; this would result in less reliable pressure-drop predictions. The relatively low values for the standard deviations for both methods given in Figure 8 reflect the selection of the data for testing the correlations; that is, the data were evaluated in order to ensure that the flow was definitely established and in the slug-flow regime.

CONCLUSIONS

The Dukler-Hubbard model appears to be a useful tool for predicting two-phase pressure losses for the slug-flow regime. The model was successfully tested with pressure-drop data for single slugs by Hubbard. This paper describes an extension of Hubbard's work to predicting pressure drops for multiple slugs in long distance horizontal pipelines. The proposed approach requires accurate predictions of in situ holdup, liquid slug and film holdup, and slug frequency. In the present treatment these predictions were made through correlation based on input liquid volume and mixture Froude number. These correlations take into account pipe size and fluid properties, which makes them useful for scaleup and design purposes. The present method, which was tested with data representing a variety of fluid properties, pipe sizes, and system pressures, is roughly as reliable as the more general Dukler-Hughmark method. However, the approach used here can be further refined through improving the individual correlations for holdup and slug frequency when additional suitable data become available.

ACKNOWLEDGMENT

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NOTATION

A = pipe cross-sectional area, sq. ft.
c = constant
D = pipe diameter, ft.
f_{tp} = two-phase friction factor
g = acceleration due to gravity, ft./sec.^{2}
g_c = gravitational constant (32.2 ft.)(lb.$_m$)/(lb.$_f$)(sec.^{2})
L_p = length of pipe, ft.
L_s = slug length, ft.
N = number of slugs
N_{Fr} = mixture Froude number, dimensionless
N_{Re} = Reynolds number, dimensionless
ΔP_f, ΔP_a, ΔP_s = frictional, acceleration, and slug pressure drop, lb./sq. in.
ΔP_T = total pressure drop, lb./sq. in.
Q_L, Q_G = volumetric flow rate of liquid and gas, respectively, cu. ft./sec.
V_f = velocity of the film prior to slug pickup, ft./sec.
V_{ns} = no-slip velocity, $(Q_L + Q_G/A)$, ft./sec.
V_p = velocity of a liquid particle, ft./sec.
V_t = translational velocity of a slug, ft./sec.
X = liquid pickup rate, lb./sec.

Greek Letters

μ_L = liquid viscosity, lb./(ft.)(sec.)
η_f, η_s, η = film, slug, and total in situ liquid holdup, volume fraction
ρ_L, ρ_s = density of the liquid and the slug, respectively, lb./cu. ft.
ν_s = slug frequency, sec.$^{-1}$
λ = input liquid content, volume fraction

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